# Notes to Presentation

* Transition from Johannes‘ Part
* Discretization
  + Now we come to the discretization of the model
  + First of all: Why do we discretize at all?
    - We want to determine the optimal control of the car with respect to the mileage and the distance that we drive. The combination of those two has to be optimal.
    - The optimal control is in our case the acceleration by the torques of the car axes and the deceleration torque

Also we want to determine the position x and the velocity v which the car has with this optimal control

* + - Now the difficult part gets into play: There is no way to determine the states and controls for the continuous optimal control problem since we would have an **infinite dimensional optimization problem**
    - So we approximate the problem by discretizing it on a grid such that we can determine the variables at every grid point.
  + How did we discretize?
    - We have a starting time by default t=0 and an end time of the optimization t\_f
    - Then we choose a number of intervals, e.g. let it be n, thus we have n+1 grid points
* Multiple Shooting
  + We have the continuity constraint that the position x and the velocity v of the solution should be at least continuous such that we do not have jumps in the position or velocity which would after all not model realistic behaviour of a car
  + We now have a discretized model so we do not know the position and velocity values at the grid point such that the trajectories are continuous
  + This continuity condition can be obtained by multiple shooting
    - The Idea of multiple Shooting for one state, e.g. velocity v, is that we use some initial velocity for the grid points and iteratively compute new values for v. This is done by solving an ordinary differential equation with those initial values with a solver like Euler or Runge Kutta. Thus we ‘shoot’ the old velocity to the next grid point having a new velocity value.

The goal is to after all have the initial of the i+1-th grid point to be equal to the shooted i-th grid point: v\_i(t+1) = v\_{i+1} (an Tafel schreiben)

and the same for position x.

* + Thus multiple shooting yields continuous trajectories of the position and velocity
  + Also one small remark we assume that the control is constant in each time step and thus changes not continuously which is not important for our model
* Results
  + In the following I’d like to show you some results of the multiple shooting
  + In this simple example the parameters are omitted, we only have the 2 states position and velocity and the 2 control acceleration- and deceleration torque
  + We look at the multiple shooting of position and velocity. We chose the initial x values to be all zero and the multiple shooting shooted them to way higher values because those were bad approximations

But the initial v seems to be close to continuous already just the second grid point seems to be a little bit off

* + In the following plots the x and v values will change due to the optimization of the model.

In this example we wanted the car to have almost 0 velocity in the beginning drive the maximal distance within 120 seconds and come to a halt in the end again. The mileage minimization has little weight compared to the maximization of the distance in this example

We discretized the problem here to 5 intervals

* + Especially for x the shooted values approach the initial values of the next grid point.

v just changes the shape due to the optimization.

* + This is the result of the multiple shooting with the optimal solution of our problem

Of course x and v are smooth since this is one of the constraints. Also the other conditions that it starts with almost 0 velocity and comes to a halt in the end are fulfilled

* + The torques also look reasonable. It makes sense that the car only tried to come as far as possible while considering only little mileage, which is why the car isn’t acceleration all the way through